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Damped vibrations of a linearly tapered homogeneous orthotropic circular plate

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Abstract : The effect of damping on free vibrations of an orthotropic homogeneous elastic circular plate of linearly varying thickness has been analysed in present research work The governing differential equation of motion has been solved by Frobenius method The frequencies corresponding to the first two modes of vibrations have been obtained for an orthotropic circular plate with different combinations of boundary conditions for various values of damping constant and taper constant

Keywords : Orthotropic circular plate, damped vibration, taper constant

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1 Introduction

For more than last two decades, interest has been highly developed in the effect of damping and temperature on solid bodies because of rapid development in space technology, high speed atmospheric flights *etc* In mechanical systems where certain parts of machine have to operate under damping, its effect is far from negligible

The engineering materials are of three types in terms of elastic symmetry *viz* isotropic, anisotropic and orthotropic type The isotropic material has an infinite number of symmetry *i e.* every plane is plane of symmetry and it requires only two elastic constants for its characterization On the other hand, a material without any plane of symmetry is called fully anisotropic it requires 21 independent elastic constants for its characterization Finally, orthotropic materials are special case of anisotropic materials. By definition, an orthotropic material has two orthogonal planes of symmetry where material properties are independent of direction within each plane such materials requires 9 independent elastic constants for their characterization These studies have

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been intensively made by Timoshenko and Krieger [1] and Ghosh [2] Several authors [3–5] have studied the natural frequency and free vibration on some circular plate Non-linear interactions in asymmetric vibrations of a circular plate was studied by Lee and Yeo [8] Gupta and Goel [12] studied the forced asymmetric response of linearly tapered circular plates Singh and Saxena [13] studied the transverse vibration of a quarter of a circular plate with variable thickness Celep [15] has studied the free vibration on some circular plate on arbitrary thickness Mc Nitt [18] introduced damping factor in free vibration of a damped elliptical plate The analysis presented here pertains to the damping effect on frequencies of a circular orthotropic homogenous plate of linearly varying thickness with different boundary conditions The first two modes of vibrations with clamped and simply supported edge conditions for various values of damping constant and taper constant have been derived

2. Theory and computation

The axisymmetric motion of a circular plate of radius 'a', in polar coordinates $(r\,\theta)$ is governed by the following equations

$$(rQ_{r,r}) = \rho h w{,}tt, \tag{1 1}$$

and

$$Qr = [(rM_r),r - M_\theta]/r, \tag{1 2}$$

where h is thickness of the plate, ρ is the mass density per unit volume of the plate w the transverse deflection, t is the time, Q_r is the shears resultant, M_r and M_θ are the movement resultants A comma followed by a suffix denotes partial differentiation with respect to that variable

The moment resultants M_r and M_θ for homogeneous and polar orthotropic material of the plate, are given by [17]

$$\begin{aligned} M_r &= -D_r\left[W{,}rr + (v_\theta / r)W{,}r\right] \\ &\text{and} \\ M_\theta &= -D_\theta\left[(1/r)W{,}r + v_r W{,}rr\right], \end{aligned} \tag{1 3}$$

where D_r and D_θ are the flexural rigidities in r- and θ directions respectively

The equation for transfer motion of a polar orthotropic circular plate of variable thickness of damping effect has been obtained from Eqs (1 1), (1 2) and (1 3) as

$$\begin{aligned} &D_r W{,}rrrr + 2[\{D_r + rD_{r,}r\}/r]W{,}rrr \\ &\quad +[\{-D_\theta + r(2 + v_\theta)D_r{,}r + r^2 D_r{,}rr\}/r^2]W{,}rr \\ &\quad +[\{D_\theta - rD_\theta{,}r + r^2 v_\theta D_r{,}rr\}/r^2]W{,}r \\ &\quad + \rho h W{,}tt + KW{,}t = 0, \end{aligned} \tag{1}$$

introducing the following non-dimensional variables

$H = h/a,\ W = \bar{w}/a,\ R = r/a,\ D_R = D_r/a^3$ and $D_\theta = D_\theta/a^3$.

$D_R = E_{11}H^3/12,\ D_\theta = E_{22}\ H^3/12,$

$E_{11} = E_1/(1 - \nu_r\nu_\theta),\ E_{22} = E_2/(1 - \nu_r\nu_\theta).$

Eq (1) can be written as

$$D_r\frac{\partial^4 W}{\partial R^4} + 2\left(\frac{D_R}{R} + \frac{\partial D_R}{\partial R}\right)\frac{\partial^3 W}{\partial R^3},\quad \frac{D_\theta}{R^2} \pm \frac{(2+\nu_\theta)}{R}\frac{\partial D_R}{\partial R} + \frac{\partial^2 D_R}{\partial R^2}$$

$$\left\{\frac{D_\theta}{R^3} - \frac{1}{R^2}\frac{\partial D_\theta}{\partial R} + \frac{\nu_\theta}{R}\frac{\partial^2 D_R}{\partial R}\right\}\frac{\partial W}{\partial R} + \rho a^2 H\frac{\partial^2 W}{\partial t^2} + K\frac{\partial W}{\partial t} = 0 \tag{2}$$

3. Solution

The solution of eq. (2) we assume

$$W = \bar{w}(r)e^{\gamma t}\cos pt \tag{3}$$

and thickness variation of the plate as

$$H_{(R)} = H_0(1-\beta R),$$

On substitution of eq. (3) into eq. (2) the Differential Equation takes the form.

$$(1-\beta R)^4\frac{\partial^4 W}{\partial R^4} + \frac{2}{R}(1-\beta R)^4\frac{\partial^3 W}{\partial R^3} - 6\beta(1-\beta R)^3\frac{\partial^3 W}{\partial R^3} - \frac{1}{R^2}\frac{E_2}{E_1}(1-\beta R)^4\frac{\partial^2 W}{\partial R^2}$$

$$+\frac{6\beta}{R}(1-\beta R)^3\frac{\partial^2 W}{\partial R^2} - 3\beta\frac{\nu_\theta}{R}(1-\beta R)^3\frac{\partial^2 W}{\partial R^2} - 6\beta(1-\beta R)^2\frac{\partial^2 W}{\partial R^2}$$

$$+\frac{1}{R^3}\frac{E_2}{E_1}(1-\beta R)^4\frac{\partial W}{\partial R} - \frac{3\beta}{R^2}(1-\beta R)^3\frac{E_2}{E_1}\frac{\partial W}{\partial R} - 6\beta^2\frac{\nu_\theta}{R}(1-\beta R)^2\frac{E_2}{E_1}\frac{\partial W}{\partial R}$$

$$-(1-\beta R)^2\Omega^2 I^* W - D_k^2 I^{*2} W = 0, \tag{4}$$

where

$$\Omega^2 = \frac{12(1-\nu_\theta\nu_r)\rho a^2 p^2}{E_1}$$

$$I^* = 1/H_0^2, \tag{5}$$

$$D_k^2 = \frac{3(1-\nu_r\nu_\theta)K^2}{E_1\rho},$$

$p \rightarrow$ Circular frequency, $\Omega \rightarrow$ frequency parameter, $D_K \rightarrow$ Damping parameter. Now the solution of eq. (4) is taken as

$$W = \sum_{K=0}^{\infty} a_k R^{C+K},\ a_0 \neq 0, \tag{6}$$

where C is exponent of singularity .

Substituting (6) in (4) and putting $E_2/E_1 = l^2$, $\nu_\theta = m^2$, we get

$$\sum_{K=0}^{\infty} a_k\left[T_1^1 b_K(3)+T_1^2 b_K(2)+T_1^3 b_K(1)+T_1^4 b_K\right]R^{C+K-4}$$

$$+\sum_{K=0}^{\infty} a_k\left[T_2^1 b_K(3)+T_2^2 b_K(2)+T_2^3 b_K(1)+T_2^4 b_K\right]R^{C+K-3}$$

$$+\sum_{K=0}^{\infty} a_k\left[T_3^1 b_K(3)+T_3^2 b_K(2)+T_3^3 b_K(1)+T_3^4 b_K\right]R^{C+K-2}$$

$$+\sum_{K=0}^{\infty} a_k\left[T_4^1 b_K(3)+T_4^2 b_K(2)+T_4^3 b_K(1)+T_4^4 b_K\right]R^{C+K-1}$$

$$+\sum_{K=0}^{\infty} a_k\left[T_5^1 b_K(3)+T_5^2 b_K(2)+T_5^3 b_K(1)+T_5^4 b_K+T_5^5\right]R^{C+K}$$

$$+\sum_{K=0}^{\infty} a_k\left[T_6^1\right]R^{C+K+1}$$

$$+\sum_{K=0}^{\infty} a_k\left[T_7^1\right]R^{C+K+2}=0$$

Eq. (7) can be written as

$$\sum_{K=0}^{\infty} a_k F_1^{(K)} R^{C+K-4}+\sum_{K=0}^{\infty} a_k F_2^{(K)} R^{C+K-3}$$

$$+\sum^{\infty} a_k F_3^{(K)} R^{C+K-2}+\sum^{\infty} a_k F_4^{(K)} R^{C+K-1}$$

$$+\sum_{K=0} a_k F_5^{(K)} R^{C+K}+\sum_{K=0}^{\infty} a_k F_6^{(K)} R^{C+K+1}$$

$$+\sum_{K=0} a_k F_7^{(K)} R^{C+K+2}=0. \tag{8}$$

For eq. (6) to be the solution, the coefficients of various powers of R in the expression obtained by substituting (6) in (8) must be identically zero. Thus by equating the coefficients of the lowest power of R to zero, one get the identical equations :

$a_0F_1(0) = 0$ since $a_0 \neq 0$ *i.e.* $F_1(0) = 0$,

$F_1(0) = T_1^1 b_0(3) + T_1^2 b_0(2) + T_1^3 b_0(1) + T_1^4 b(0) = 0$,

$[C(C-1)(C-2)(C-3)] + 2[C(C-1)(C-2)] - l^2[C(C-1)] + l^2 C = 0$,

$C = 0,\ 2,\ 1+l,\ 1-l$.

It may be seen that the series corresponding to $C = 0$ will also contain the series corresponding to $C = 2$. Also, the series corresponding to $C = 1-l$, $l > 1$ vanishes because of its singularity at $R = 0$ and the series for $C = 1-l$, $l < 1$ will be contained in the series corresponding to $C = 1+l$.

Further equating the coefficient of the next subsequent power of R to zero, it is found that $a_1 = 0$, a_2 is indeterminant, so we can choose any value of a_2 and $a\lambda$ ($\lambda = 3, 4, 5......$) can be written in terms of a_0 and a_2.

Hence assuming

$$a_\lambda = A_\lambda a_0 + B_\lambda a_2 (\lambda = 0,1,2,3.........), \tag{9}$$

the following solution, corresponding to $C = 0$ and $C = 1+l$ is obtained :

$$W = a_0 \left| 1 + \sum_{\lambda=3}^{\infty} A_\lambda R^\lambda \right. + a_2 \left| 1 + \sum^{\infty} B_\lambda R^{\lambda+1+l} \right. \tag{10}$$

It is also evident that the solution corresponding to the other value of C is induced in (10) Hence no new solution can exist for this value of C.

Application of the test of Lamb [19] shows that the solution (10) is convergent for $\beta < 1$, $|\mu|<1$.

4. Boundary conditions and frequency equations

The following combinations of boundary conditions have been considered :

(i) Clamped (C) at the edge $R = 1$.

(ii) Simply supported (S) at the edge $R = 1$.

The boundary conditions for different edge conditions are :

For clamped edge

$$W = 0 \text{ and } \frac{\partial W}{\partial R} = 0. \tag{11}$$

For simply supported edge,

$W = 0$ and $M_R = 0$,

i.e.

$$W = 0 \text{ and } \frac{\partial^2 W}{\partial R^2} + \frac{1}{R} v_\theta \frac{\partial W}{\partial R} = 0. \tag{12}$$

C-Plates :

After applying boundary conditions, the frequency equation for clamped plate is

$$\begin{vmatrix} F(1,\lambda^2) & G(1,\lambda^2) \\ F^1(1,\lambda^2) & G^1(1,\lambda^2) \end{vmatrix} = 0. \tag{13}$$

S-Plates :

Applying boundary condition, the frequency equation is

$$\begin{vmatrix} F(1,\lambda^2) & G(1,\lambda^2) \\ F_1(1,\lambda^2) & G_1(1,\lambda^2) \end{vmatrix} = 0, \tag{14}$$

where

$$F_1(1,\ \lambda^2) = F'(1,\ \lambda^2) + \nu_\theta F'(1,\ \lambda^2$$

and

$$G^1(1,\ \lambda^2) = G\ (1,\ \lambda^2) + \nu_\theta G\ (1,\ \lambda^2)$$

Here, dash denotes the partial differentiation with respect to R.

5. Result and discussion

Frequency equations (13) and (14) are transcendental equations in λ^2 from which infinitel roots can be determined The frequency parameter λ corresponding to the first two modes of vibrations of a clamped and simply supported orthotropic circular plate has been computed for different values of damping constant and taper constant The orthotropic materials are taken as [14]

$$l^2 = 1.44 \text{ and } m^2 = 0.3.$$

It is concluded that the frequencies in the first two modes of vibration decrease with the increasing values of damping constant as well as taper constant in both the cases of boundary conditions, considered here. The frequencies corresponding to the first two modes of vibration for various values of damping constant (D_K) and taper constant (β for both the boundary conditions have been plotted in Figures 1 and 2 For comparing the numerical values of the frequency with those of Tomar and Tiwari [14], λ has also been

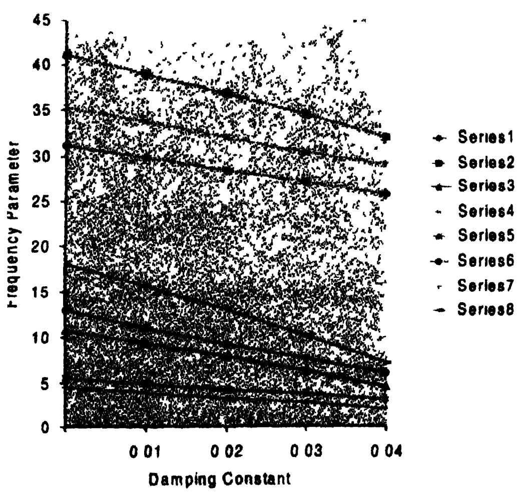

Figure 1. Variation of frequency parameter with damping constant for a circular plate of linearly varying thickness (colour on line).

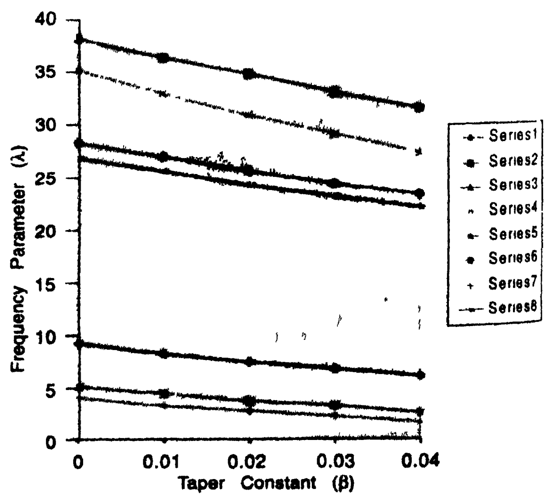

Figure 2. Variation of frequency parameter with taper constant for a circular plate of linearly varying thickness (colour on line).

computed for the corresponding elastic constant for a clamped and simply supported orthotropic circular plate $D_K = 0 = \beta$, and it is found that the results are in satisfactory agreement for the first two modes of vibration.

Values of transverse deflection W corresponding to first two modes of vibration for C- and S-plates at different points, have been calculated for $\beta = 0.2$ and $D_K = 0.01$. These results are plotted in Figure 3 for a circular plate of linearly varying thickness.

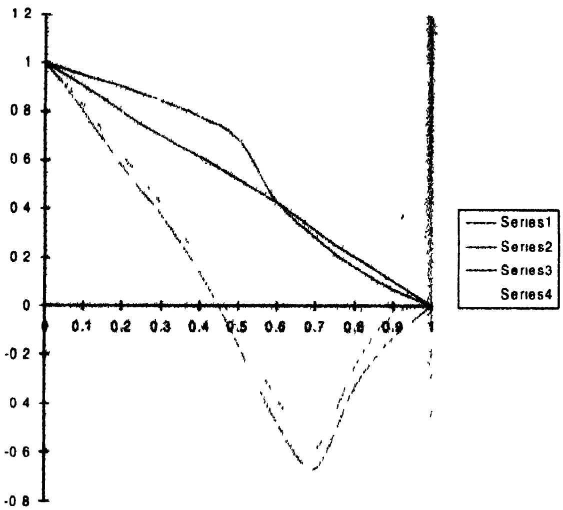

Figure 3. Transverse deflection for a circular plate of linearly varying thickness (colour on line).

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